Efficient electron heating in relativistic shocks and gamma-ray-burst afterglow

M. Gedalin,[1] M. A. Balikhin,[2] and D. Eichler[1]
[1]Department of Physics, Ben-Gurion University, Beer-Sheva, Israel
[2]ACSE, University of Sheffield, Sheffield, United Kingdom
(Received 6 March 2007; revised manuscript received 19 September 2007; published 7 February 2008)

Electrons in shocks are efficiently energized due to the cross-shock potential, which develops because of differential deflection of electrons and ions by the magnetic field in the shock front. The electron energization is necessarily accompanied by scattering and thermalization. The mechanism is efficient in both magnetized and nonmagnetized relativistic electron-ion shocks. It is proposed that the synchrotron emission from the heated electrons in a layer of strongly enhanced magnetic field is responsible for gamma-ray-burst afterglows.

DOI: 10.1103/PhysRevE.77.026403 PACS number(s): 52.35.Tc, 98.70.Rz, 95.30.Qd

I. INTRODUCTION

Electron energization is usually considered as a secondary problem in heliospheric shocks, where most attention is paid to ion heating and reflection. In astrophysical shocks, however, these energized electrons emit the observed radiation, and are frequently the only source of information about the remote astrophysical process. Gamma-ray-burst (GRB) afterglow is believed to be synchrotron emission from electrons accelerated in the shock that develop during the interaction of the expanding ultrarelativistic plasma into the interstellar medium (ISM) [1]. Estimates (e.g., Ref. [2], and references therein) suggest that the required average energies of electrons reach a sizable part of the relativistic ion energy, and that the magnetic field in the emission region should be highly amplified; however, the origins of the electron heating and the magnetic field amplification remain poorly understood. In this paper we propose a single mechanism that accomplishes both, and is driven by the preferential deflection of electrons versus ions, when the former are lighter than the latter, by a local increase in the magnetic field.

The mechanism of electron heating in heliospheric shocks is widely understood as follows [3]. Electrons are decelerated more easily than ions, either by growing coherent magnetic fields in quasiperpendicular shocks or by small-scale magnetic structures in quasiparallel shocks. The developing charge separation, however small it is, results in the buildup of a cross-shock potential which is a substantial fraction of the incident ion energy. It is this cross-shock potential that decelerates ions when they become demagnetized in a thin transition layer of a quasiperpendicular shock. In quasiparallel shocks the parallel component of the magnetic field does not affect the ion motion along the shock normal, so that ions effectively become demagnetized just ahead of the transition. The same cross-shock potential that decelerates ions should accelerate electrons along the shock normal, thus transferring energy from ions to electrons. The efficiency of the process is reduced by the electron drift in the magnetic fields, during which they lose energy by drifting down an electric potential. The final step of the process, electron thermalization, can be achieved by turbulent scattering following plasma instabilities.

The mechanism of the prompt electron heating in steady-state magnetized shocks is well known [4]: electrons become demagnetized in the shock front if the ramp width is smaller than their convective gyroradius, or when the cross-shock electrostatic field becomes sufficiently inhomogeneous to drag them across the magnetic field. In heliospheric shock these conditions are rarely satisfied since shocks are rarely this narrow. Moreover, only that part of the cross-shock potential which cannot be eliminated by transformation into a de Hoffman–Teller frame [5] can be effectively used for electron energization. However, the profiles become steeper with the increase of the Mach number [6] so that the conditions for demagnetization may be achieved more easily. The transition layer of quasiperpendicular nonrelativistic shocks consists of several distinct regions [7], the steepest magnetic field increase is a "ramp" (whose width is less than the ion inertial length $l_i=c/\omega_{pi}$, $\omega_{pi}^2=4\pi n_u e^2/m_i$) and a large magnetic overshoot (whose width is of the order of the downstream ion gyroradius). The overshoot height is found experimentally to increase with increase of the Mach number [8]. The ratio of the ramp width to the ion convective gyroradius $\sim l_i \cos\theta/(V_u/\Omega_u) \sim 1/M$, where θ is the angle between the shock normal and the upstream magnetic field $\Omega_u=eB_u/m_i c$ is the upstream ion gyrofrequency, and $M=\Omega_u/\omega_{pe}$ is the Alfvénic Mach number. In perpendicular shocks the ramp width can be as small as $l_e=c/\omega_{pe}$ [9].

The theory of electron heating in quasiparallel shocks has been developed less elaborately, partly because of the lack of coherent structure in these shocks. Observations [10] imply that the dominant electron heating process is the same as in quasiperpendicular shocks and appear to illustrate the importance of the dc effects of the coherent forces for the physics of electron heating in shocks.

GRB-generated forward shocks in the ISM are ultrarelativistic $\Gamma \geqslant 20$. These shocks are parametrized by $\sigma = B_u^2/4\pi n_u m_i c^2 \gamma_u \ll 1$ (this is written in the shock frame but is invariant). They are very high Mach number shocks, since the corresponding Mach number $M=1/\sigma$. Based on numerical simulations, it is widely believed that such shocks may be formed due to the development of a Weibel instability [11] into ion current filaments surrounded by regions of enhanced magnetic field. The filaments are elongated along the flow direction, with the magnetic field nearly perpendicular to the shock normal. The magnetic field around the filaments reaches nearly equipartition values but the magnetic filling factor is low. The width of a magnetic region is expected to be up to tens of the electron inertial length while the length of the region over which the surrounding magnetic field is high is determined by the ion scale. Although there is no gyration in these structures, high magnetic fields at small scales make them play the role of a perpendicular magnetized shock front in what concerns electron energization.

In this paper we suggest that differential momentum transfer to ions and electrons, typical for steady perpendicular shock and filamentary shock as well, results in the buildup of a strong potential drop, comparable to the upstream ion energy. The electrons are demagnetized and receive a significant fraction of the original ion kinetic energy directly from the dc electric field. The accelerated electron energy is converted into either gyration energy (by the coherent magnetic field in magnetized shocks) or random motion energy (by small-scale magnetic fields in Weibel-mediated shocks) thus resulting in collisionless heating. In both cases a region of strongly enhanced magnetic field is developed in the shock front, where the heated electrons should efficiently emit synchrotron radiation. We show that, although the details of the mechanism differ in magnetized and nonmagnetized shocks, the underlying physics is very similar, and the eventual efficiency does not depend on the magnetization. GRB afterglows may be explained, at least in part, by radiation from these heated electrons.

In proposing a mechanism for electron heating based on charge separation we do not mean to deny the existence of other mechanisms, e.g., decay and merging of magnetic islands, which can operate even with equal masses of both species. However, because the Weibel shock is otherwise required to "wait" for a bootstrap process in which electrons are heated by magnetic field, but magnetic field growth is limited by electron temperature [12], we suggest that in the case of realistic mass ratios even a modest degree of charge separation can help to jump start the collisionless shock process.

II. MAGNETIZED SHOCKS

As will be seen below, magnetized shocks are more restrictive in producing efficient electron heating, yet the basic features of the mechanism are typical for nonmagnetized shocks as well (with suitable modifications). Therefore, we start our analysis with quasiperpendicular magnetized shocks.

Relativistic shock propagating obliquely in the ISM becomes nearly perpendicular in the shock frame, because of the Lorentz transformation, $\theta_{\rm shock}=\theta_{\rm ISM}/\gamma_u \ll 1$ (here $\gamma_u \gg 1$ is the Lorentz factor of the shock relative to the ISM or, alternatively, the Lorentz factor of the incident plasma flow in the shock frame). The de Hoffman–Teller frame, which has the velocity $V_u \tan\theta$ along the shock front, does not exist for $V_u \approx c$ and $\tan\theta > 1/\gamma_u$. In what follows we consider first a quasistationary perpendicular magnetized shock front where the fields are given by $B_z=B(x)$, $E_x(x)$, and E_y =const.

A. Demagnetization conditions

The condition for the demagnetization by inhomogeneous E_x is the statement that the accelerating electric field straightens the trajectory faster than the magnetic field bends it. The condition can be derived in the simplest way by approximating the inhomogeneous electric field with a linear slope while ignoring the magnetic field variations in the electron equations of motion. Then the motion is described by $\mathbf{v}-\mathbf{v}_0$, $x-x_0 \propto \exp(\lambda t)$. Imaginary λ ($\lambda^2<0$) corresponds to the particle gyration in the magnetic field (magnetic bending prevails) while $\lambda^2>0$ results in the exponential acceleration across the magnetic field, that is, demagnetization [4]. Relativistic generalization of the calculations in Ref. [4] is straightforward (see Appendix B) and gives

$$-\gamma(1+\gamma^2 v_y^2/c^2)(e/m_e)\frac{dE_x}{dx} > \Omega_e^2, \qquad (1)$$

where $\Omega_e = eB/m_e c$. If (1) is satisfied, electrons are efficiently accelerated across the magnetic field and acquire most of the cross-shock potential at the demagnetization region. The condition is local and cannot be satisfied in the whole shock transition layer, since $-dE_x/dx>0$ is required. Thus, the electrons can be demagnetized while crossing a part of the magnetic inhomogeneity, after which they may return to be magnetized and the acquired energy is immediately converted into their gyration energy. Alternatively, electrons become demagnetized if the inhomogeneity scale of the magnetic field $(1/B)(dB/dx)$ is smaller than the convective electron gyroradius $c\gamma_e/\Omega_e$.

The above demagnetization condition is derived in a simplified assumption that the magnetic field is constant. While this is not the case inside the shock, numerical analyses [4] have shown remarkable agreement with application of the nonrelativistic version of (1) at the upstream edge of the ramp, and (1) should be considered an estimate.

Demagnetization is required for an electron to utilize the cross-shock potential, otherwise electrons simply $\boldsymbol{E}\times\boldsymbol{B}$ drift, and the energy gain due to the potential (E_x) is balanced by the energy loss because of the motion along E_y. Once the drift is substantially suppressed a net energy gain is achieved [4]. The energy gain is determined by the potential drop across the demagnetization region. When magnetization is restored no further energization occurs. The acquired energy is converted into the electron gyration energy where demagnetization disappears. Further collisionless "randomization" occurs through gyrophase mixing in the nonstationary and inhomogeneous fields of the shock front, thus resulting in collisionless heating [3]. Maxwellization is not required for the existence of the shock.

B. Magnetic structure and cross-shock electric field

For the purpose of description we consider a one-dimensional and steady shock. The basic equations of the two-fluid hydrodynamics for this shock are given in Appendix A. The cross-shock electric field can then be estimated using the momentum conservation

$$\sum T_{xx} + \frac{B^2-E^2}{8\pi} = \text{const}, \quad T_{xx} = \langle p_x v_x \rangle. \qquad (2)$$

Here $\langle\cdots\rangle$ means averaging over the distribution function and the summation is over both species. The discussion below is based on the basic picture, justified by observations [6], simulations [13], and theory [14], that the front steepening stops at a width much smaller than the convective ion gyroradius which ensures ion demagnetization inside the shock transition layer, and the assumption that this basic picture applies to relativistic magnetized shocks. As a consequence, ions are only slightly deflected within the transition layer (ramp) while almost all current necessary for the magnetic field increase is produced by electrons, which (partially) experience $\boldsymbol{E}\times\boldsymbol{B}$ drift. The latter allows one to estimate the electron velocity as $v_y \sim (c/4\pi n_u e)(dB_z/dx)$. Before electrons are substantially heated the magnetic force should be balanced by the electric force so that

$$E_x \approx -\frac{1}{8\pi n_u}\frac{d}{dx}B_z^2, \quad \Rightarrow e n_u \Delta\phi \approx \Delta B^2/8\pi, \tag{3}$$

where we have taken into account approximate quasineutrality and neglected the change of the ion density. For $\sigma \ll 1$ even slight deceleration of ions causes strong enhancement of the magnetic field, which results in the development of the cross-shock potential which, in turn, further decelerates ions. A spontaneous small enhancement of the upstream magnetic field causes exponential development of the magnetic field increase at the typical electron length scale (see below). The corresponding electric field given by (3).

Upon crossing this narrow region of the magnetic field increase and potential development ions begin to gyrate. Assuming the gyrating ions to be a cold beam, it is easy to see that the momentum flow T_{xx} in the particles is very small where the ions have gyrated by 90° and are moving nearly perpendicularly to the flow (x) direction. If the shock is to be quasistationary, this must be taken up by some combination of magnetic and electron pressure. For a weakly magnetized shock, magnetic pressure balance would imply a magnetic field far larger than that dictated by shock jump conditions. Electron pressure would require significant cross-shock potential. The two quantities are connected by Eq. (3), so the argument implies both magnetic overshoot and a large cross-shock potential. Since the shock may be unsteady, this argument does not constitute a rigorous proof of either; however, it shows that ion reflection is likely to cause extremely chaotic conditions in which pressure balance without strong cross-shock potential and magnetic overshoot would seem to require implausibly fine tuning.

In order to know whether electrons are indeed demagnetized one has to know the spatial profile of the shock. Two-fluid hydrodynamics predicts [14] that a perpendicular magnetosonic wave steepens down to the slope determined by the electron inertial length l_e. Following the general principles of [14], we seek nonlinear wave solutions that are asymptotically homogeneous, that is, $n\to n_0$, $v_x\to v_0$, $B_z\to B_0$, $v_y\to 0$, when $x\to -\infty$. In this case $E_y=v_0B_0/c$. In the usual quasineutrality approximation charge separation is weak throughout the wave profile $\delta n=(1/4\pi e)(dE_x/dx)\ll n$. Further derivation is given in Appendix C and results in the equation

$$\left(\frac{c^2}{\omega_{pe}^2}\right)\frac{1}{N}\frac{d}{dx}\frac{\gamma_e}{N}\frac{db}{dx} = \frac{(1+\sigma)(b-1)-\sigma b(b^2-1)/2\beta_0^2}{1-\sigma(b-1)}, \tag{4}$$

where $N=n/n_0=v_0/v_x$, $N=[1-\sigma(b-1)][1-\sigma(b^2-1)/2\beta_0^2]^{-1}$, and $b=B/B_0$. The obtained expression is similar to those obtained previously for nonlinear stationary waves in pair plasmas [15]. It is easy to see that the equation predicts the slope scale of $\tilde{l}_e=c\sqrt{\gamma_e}/\omega_{pe}$. The ratio $\tilde{l}_e/r_e=(m_e/m_i)^{1/2}\sqrt{\gamma_e}\sigma^{-1/2}\ll 1$ for typical parameters of gamma-ray bursts. Therefore, electrons are expected to be demagnetized. It has to be understood, however, that the above small scale requires corresponding electron drift along the shock normal to ensure the current necessary to sustain the slope. Trajectories of demagnetized electrons are straightened along the shock normal and their drift is substantially suppressed, so that the ramp steepening does not proceed to scales much smaller than those required by the demagnetization condition. From the expression for N and Eq. (4) one can see that the amplitude of the magnetic compression reaches the values $b\sim 1/\sqrt{\sigma}$ for strongly nonlinear structures in a low-σ plasma, in agreement with the estimates made independently earlier in this paper.

To summarize, the basic points are the following: (a) electrons become demagnetized if the typical inhomogeneity scale becomes smaller than the electron convective gyroradius $c\sqrt{\gamma_e}/\omega_{pe}$ or the cross-shock electric field slope is sufficiently steep to satisfy (1), whichever happens first; (b) the cross-shock electric field E_x is related to the magnetic field as in (3), so that the potential increases with B^2; (c) the magnetic field, and hence the cross-shock potential, increases to high values because magnetic pressure has to compensate the decrease of ion T_{xx} as described by (2); (d) large-amplitude magnetosonic waves steepen down to the scales $c\sqrt{\gamma_e}/\omega_{pe}$, as described by (4), which follows directly from the assumptions of electron drift and quasineutrality; (e) the magnetic field in these structures increases up to $B/B_u\sim 1/\sqrt{\sigma}$ before the singularity $v_x=0$ is reached; (f) according to (3) the cross-shock potential is a substantial part of the incident ion energy; and (g) the estimates above show that electrons have to be demagnetized [width is less than their convective gyroradius or (1) is satisfied]. While not constituting a rigorous proof, these arguments show the plausibility and self-consistency of the proposed scenario of electron demagnetization by inhomogeneous cross-shock electric field and consequent heating. While the above scenario is described in terms of a monotonic magnetic field and potential increase across the ramp, it is likely that in real shocks the ramp itself breaks into substructures and the electron heating occurs as a series of electric spikes [16].

III. NONMAGNETIZED SHOCKS

Nonmagnetized shocks are characterized by a very weak (or zero) upstream magnetic field so that the upstream convective gyroradii of both species exceed the system size and coherent magnetic braking is impossible. Weibel instability [11] produces magnetic filaments ahead of the main transition [17]. Strong electron heating appears to be necessary for Weibel mediation at $\sigma\leqslant\eta(T_e/m_ic^2)^3$ [12], where η is a dimensionless number less than unity. Otherwise, Weibel turbulence is predicted to be rather small scale and weak, so that ion scattering is relatively inefficient. Small-scale magnetic filaments, where the magnetic field is aligned perpendicular to the shock normal in tubes or sheets, scatter forward-going electrons more readily than ions (as does the perpendicular magnetic field enhancement for magnetized shocks), even if the electrons are not fully magnetized. Any deflection reduces the speed along the shock normal, thus the inflowing electrons are slowed relative to the inflowing ions by the turbulent field. The structure is no longer one dimensional and stationary so that (A5) and (A6) are not applicable directly and (A1) and (A2) should be used. However, assuming that electrons are scattered essentially randomly but small-scale fields of the filaments, and neglecting ion scattering, one can average the equations over the perpendicular dimensions and time scales smaller than the ion transit time. Let us consider a single-particle motion in the filamentary structure, taking the latter as given. The equations of motion read (for any species)

$$\frac{d}{dt}p_x = qE_x + q\hat{\boldsymbol{x}} \cdot (\boldsymbol{v}_{\text{tr}} \times \boldsymbol{B}_{\text{tr}}), \tag{5}$$

$$\frac{d}{dt}\boldsymbol{p}_{\text{tr}} = q\boldsymbol{E}_{\text{tr}} + qv_x(\hat{\boldsymbol{x}} \times \boldsymbol{B}_{\text{tr}}), \tag{6}$$

where "tr" denotes $\perp\hat{\boldsymbol{x}}$. Here we assume that $\boldsymbol{E}_{\text{tr}}$ and $\boldsymbol{B}_{\text{tr}}$ are small-scale rapidly (in space and time) fluctuating fields [17], while E_x contains a global coherent electric field also. Denoting by an overbar averaging over rapid fluctuations, we assume that $\bar{\boldsymbol{E}}_{\text{tr}}=\boldsymbol{0}$, $\bar{\boldsymbol{B}}_{\text{tr}}=\boldsymbol{0}$, $\bar{\boldsymbol{v}}_{\text{tr}}=\boldsymbol{0}$, but $\bar{E}_x \neq 0$, $\bar{v}_x \neq 0$, $\overline{E^2_{\text{tr}}} \neq 0$, $\overline{B^2_{\text{tr}}} \neq 0$, and $\overline{v^2_{\text{tr}}} \neq 0$. In the lowest-order approximation the particle flow is along x and scattering can be treated perturbatively:

$$\boldsymbol{p}_{\text{tr}} \approx [q\boldsymbol{E}_{\text{tr}} + qv_x(\hat{\boldsymbol{x}} \times \boldsymbol{B}_{\text{tr}})]\tau, \tag{7}$$

where τ is a characteristic "collision" time. Approximating $v_x \approx c$, substituting (7) into (5) and averaging over rapid fluctuations, one has

$$\bar{v}_x \frac{d}{dx}\bar{p}_x = q\bar{E}_x + \frac{q^2\tau}{m\gamma}[\hat{\boldsymbol{x}} \cdot \overline{(\boldsymbol{E}_{\text{tr}} \times \boldsymbol{B}_{\text{tr}})} - \overline{B^2_{\text{tr}}}], \tag{8}$$

which is written for ions and electrons as well. Here we substituted $(d/dt) \rightarrow \bar{v}_x(d/dx)$.

Simulations [11] show that the generated magnetic field patterns are advected toward the shock front at speeds intermediate between those of the incoming plasma and the rest-frame plasma. In this case the electric fields are substantially weaker than the magnetic fields in the shock frame, so that the $\hat{\boldsymbol{x}} \cdot \overline{(\boldsymbol{E}_{\text{tr}} \times \boldsymbol{B}_{\text{tr}})}$ term can be neglected relative to the last term, which is nothing but the magnetic braking due to filaments. We now involve the smallness of τ expected from the Weibel instability. The fastest-growing modes have a scale length between the electron and ion inertial lengths [12]. This means that, in considering electron scattering, which we propose as a physical origin of charge separation, the ion scattering term which is proportional to τ/m_i is small relative to the electron scattering term which is proportional τ/m_e. Thus, while the two terms on the right-hand side of the electron equation (8) may be comparable for electrons, the last term is neglected for ions. Therefore, the ion motion is described by

$$\bar{v}_{i,x}\frac{d}{dx}\bar{p}_{i,x} = e\bar{E}_x \tag{9}$$

and, for $\bar{v}_{i,x}=c$ (negligible scattering of relativistic ions), one has

$$c\Delta\bar{p}_{i,x} = -e\Delta\phi, \quad \phi = -\int \bar{E}_x dx, \tag{10}$$

in complete analogy with what happens to ions in a magnetized shock ramp: ions are decelerated by the potential which builds up due to charge separation caused by more efficient magnetic braking of electrons.

With the same approximation, the electron energy changes as follows:

$$\frac{d}{dt}(m_e c^2 \bar{\gamma}_e) = -e\overline{E_x v_x} - e\overline{\boldsymbol{E}_{\text{tr}} \cdot \boldsymbol{v}_{\text{tr}}} \tag{11}$$

$$\approx -e\bar{E}_x\bar{v}_x + e^2\frac{q\tau}{m_e\gamma_e}[\overline{E^2_{\text{tr}}} - \hat{\boldsymbol{x}} \cdot \overline{(\boldsymbol{E}_{\text{tr}} \times \boldsymbol{B}_{\text{tr}})}]. \tag{12}$$

Unless the last term just happen to cancel the first term on the right-hand side, the electrons acquire energy which is of the order of the potential drop $e\Delta\phi$. Since this is the potential that decelerates ions, $e\Delta\phi \sim m_i\gamma_0 c^2$, therefore,

$$\Delta(m_e c^2 \bar{\gamma}_e) \approx e\Delta\phi \sim m_i\gamma_0 c^2, \tag{13}$$

so that electrons acquire energy comparable to what the ions lose. Although we have not rigorously proved that this cancellation is impossible, we may note that in a highly turbulent nonlinear environment the second term is likely to be a highly erratic function of space and time, and it does not seem likely that its average would cancel the first term. That the first term should be of significant size is based on the fact that electrons are more easily scattered than the ions by the electromagnetic turbulence, and this naturally results in systematic charge separation during the early stages of a Weibel shock. The efficiency of energy transfer is higher than in magnetized shocks, where only about one-half of the potential can be acquired by electrons. This is because the electrons remain almost completely demagnetized throughout the whole region where ions decelerate. Yet the electrons do not acquire all the momentum lost by ions, because of their scattering. Part of the momentum is transferred to the electromagnetic field. The pressure balance in this case takes the form

$$\sum \langle p_x v_x \rangle + \frac{\overline{B^2_{\text{tr}}} + \overline{E^2_{\text{tr}}} - \overline{E^2_x}}{8\pi} = \text{const}. \tag{14}$$

Simulations [11,17] show that filaments are convected by plasma and merge, so that both the local and average magnetic field density increase toward the shock transition layer. This is consistent with (14): when approaching the transition the ion momentum decreases, as well as $\overline{E^2_{\text{tr}}}$ (the latter because of the growth of the typical width of a filament), while

$\overline{B}_{\rm tr}^2$ should increase. Similarly to what happens in magnetized shocks, magnetic braking of ions is necessary to convert the energy of the directed flow into thermal energy and decelerate the ion flow down to a subrelativistic velocity. As a result, the magnetic field is expected to achieve locally the equipartition values. This is also the region where the electron scattering by the magnetic field becomes strong. Once the electrons and ions completely thermalize the magnetic pressure should drop to much lower magnitudes. A transient region of a drastic local enhancement of small-scale magnetic field forms.

Summarizing, all basic features found earlier in magnetized shocks (differential magnetic braking, buildup of a potential and electron acceleration along the shock, magnetic field increase to equipartition values, conversion of the directed flow energy into thermal energy) are also present in nonmagnetized shocks; in the latter, the local inhomogeneous magnetic fields play the role of the large-scale magnetic background of the former. The spatial scales of the corresponding "ramp" and "overshoot" are different and determined by ion gyroradius in magnetized shocks, and by the filament merging in Weibel shocks.

IV. SYNCHROTRON EMISSION

Having proposed that electrons acquire a substantial part of the incident ion energy due to the cross-shock potential prior to entering a region of strong magnetic field, we can now estimate synchrotron emission from this region. The main radiating region in magnetized shocks is the overshoot, behind which the magnetic field drops to low values. The radiating region in nonmagnetized shocks should include the filamentary region before and behind the magnetic density peak as well. The estimates below are valid for magnetized and nonmagnetized shocks as well. Let a shock propagate with the Lorentz factor γ_0 into interstellar medium with the density $n_{\rm ISM}$ and magnetic field $B_{\rm ISM}$, with $\sigma = B_{\rm ISM}^2/8\pi n_{\rm ISM} m_i c^2 \ll 1$. In the shock frame the upstream density and magnetic field are $n_u = n_{\rm ISM}\gamma_0$, $B_u = B_{\rm ISM}\gamma_0$. The electron energy in the overshoot is a fraction of the incident ion energy, that is, $\gamma_e = f_1\gamma_0/\mu$. The overshoot magnetic field is $B_o^2/8\pi = f_2 n_u m_i c^2\gamma_0$. The electron density in the overshoot follows the ion density, which remains of the same order as the upstream density, $n_e \sim n_u$. At the lower end of the energy spectrum, the electrons emit synchrotron emission with the characteristic frequency and power (in the shock frame), respectively, $\omega_m = (eB_o/m_e c)\gamma_e^2$, $P_m = (4/3)\sigma_T c\gamma_e^2(B_o^2/8\pi)$, where σ_T is the cross section of Thomson scattering. In the observer's frame the characteristic frequency is $\omega_{\rm obs} = \gamma_0\omega_m$, and the emission from unit perpendicular area becomes $(dP/dS)_{\rm obs} = \gamma_0^2 P_m N_s$, where $N_s = n_e r_o$ is the invariant surface density of electrons. Here r_o is the effective length of the radiating region. The observed frequency and emission per unit perpendicular area are

$$\omega_{\rm obs} = (8\pi e^2/m_e)^{1/2} n_{\rm ISM}^{1/2}\gamma_0^4 f_1^2 f_2^{1/2}\mu^{-5/2}, \tag{15}$$

$$(dP/dS)_{\rm obs} = 2\sigma_T m_i c^3 f_1^2 f_2\gamma_0^7 n_{\rm ISM}^2\mu^{-2} r_o. \tag{16}$$

The largest uncertainty is in r_o since there is no satisfactory theory of the relativistic shock structure (neither magnetized nor nonmagnetized). In a magnetized shock the effective overshoot width is determined by the ion gyroradius in the enhanced magnetic field, $m_i c^2\gamma_0/eB_o$, times the number of ion loops necessary for gyrophase mixing. The maximum overall length is expected to be of the order of the ion downstream gyroradius or less, that is, $r_o \lesssim f_3 m_i c^2\gamma_0/B_u$, where f_3 may be substantially smaller than unity. Correspondingly, $(dP/dS)_{\rm obs} \approx (10^9\ {\rm erg/cm^2\ s}) \times (n_1^2/B_3)\gamma_{10}^7 f_1^2 f_2 f_3$, where we normalized with the typical parameters for interstellar medium: $n_1 \equiv n_{\rm ISM}/1\ {\rm cm^{-3}}$, $B_3 \equiv B_{\rm ISM}/(3\ \mu{\rm G})$. For a typical gamma-ray burst $\gamma_0 = 10-30$ several hours after the burst, and $\gamma_{10} \equiv \gamma_0/10$. In Weibel-mediated shocks, the overshoot width is determined by the ion inertial length [17]. In this case the enhanced magnetic field is strongly inhomogeneous, so that the effective radiating width is $r_o = f_4(c/\omega_{pi})$, where f_2 and f_4 together take into account the filling factor of about 10–15 %. Simulations [17] show that in Weibel-mediated shocks the peak magnetic density region is of the width of $\sim 50(c/\omega_{pi})$, but the region where $B^2/8\pi \sim 0.1 n_u m_i c^2\gamma_0$ may be by an order of magnitude larger. The effective emission region may appear even substantially wider if the magnetic field decays as a power law [17] (see [18] for observational predictions of GRB emission in the case of decaying magnetic field). Modestly estimating for these shocks $f_1 \sim 1$, $f_2 f_4 \sim 10^2$, one finds $(dP/dS)_{\rm obs} \approx (10^6\ {\rm erg/cm^2\ s}) \times n_1^{3/2}\gamma_{10}^7$. For the isotropic equivalent emitting area $10^{34}\ {\rm cm}^2$ the total emitted power is $P \sim (10^{43}\ {\rm erg/s}) \times (n_1^2/B_3)\gamma_{10}^7 f_1^2 f_2^2 f_3$ in the magnetized case and $P \sim (10^{40}\ {\rm erg/s}) \times n_1^{3/2}\gamma_{10}^7$ for nonmagnetized shocks, emitted at the frequencies $\omega_{\rm obs} \sim (10^{17}\ {\rm s}^1) \times n_1^{3/2}\gamma_{10}^4 f_1^2 f_2^{1/2}$. In both magnetized and nonmagnetized shocks the magnetic field behind the overshoot drops down, $B_d \sim B_o\sqrt{\sigma}$. Correspondingly, the radiation frequency drops by the same factor, while the emission power drops by the factor $1/\sigma$.

This radiation from a thin region of enhanced magnetic field may be a significant fraction of the total afterglow emission. Consider the ratio of the afterglow from the magnetic region and from the entire downstream region. The fraction of the proper hydrodynamic time scale, $\tau_h \sim R/\gamma_0 c$, that an electron spends in the effective overshoot region is given by τ_o/τ_h, which is $\sim f_3 r_u/c\tau_h \sim f_3 m_i c^2\gamma_0/eB_{\rm ISM}R$ for the magnetized overshoot and $\sim f_4/\omega_{pi}\tau_h$ for nonmagnetized shocks. The ratio of the magnetic energy density in the overshoot region to the average magnetic energy downstream is $\sim 1/\sigma$. Electron energies may remain comparable due to effective turbulent collisions. The relative afterglow outputs from the overshoot region and downstream is then $\sim f_2^2\tau_o/\tau_h\sigma$, where $R \sim cT_{\rm obs}\gamma_0^2$, $T_{\rm obs}$ being the observer time, so that

$$\frac{P_{\rm overshoot}}{P_{\rm downstream}} \sim \frac{f_2^2 f_3 \times 10^{-4}}{[B/(3\mu{\rm G})][T_{\rm obs}/(10^5\ {\rm s})]\sigma} \gg 1 \tag{17}$$

for a magnetized shock and

$$\frac{P_{\rm overshoot}}{P_{\rm downstream}} \sim \frac{f_2^2 f_4 \times 10^{-8}}{n_1^{1/2}[T_{\rm obs}/(10^5\ {\rm s})]\sigma} \gg 1 \tag{18}$$

for a nonmagnetized shocks. For realistically low σ, the emission power from the enhanced magnetic field region formally exceeds the emission power in the rest of the downstream region. And the typical frequencies are much greater as well.

The cooling energy γ_c is given by the condition $P_m(\gamma_c)r_o/c \sim m_e c^2 \gamma_c$ and therefore $\gamma_c \sim \mu/\sigma_T \gamma_0^2 r_o n_{\rm ISM} f_2$ which corresponds to the cooling frequency in the observer's frame $\omega_c \sim \omega_{\rm obs}(\mu\gamma_c/f_1\gamma_0)^2 \sim (10^{18}\ {\rm s}^{-1}) \times (B_3^2 f_1^2/n_1 f_2 f_3)$ for the magnetized shocks, and much higher for demagnetized shocks, which means that radiative cooling does not affect the described processes.

V. CONCLUSIONS

We have shown above that efficient electron heating in relativistic collisionless shocks can be generated by a cross-shock potential, developing because of the preferential deflection of electrons by the magnetic field. The cross-shock potential, which accelerates electrons across the shock front, is of the order of the incident ion energy, independently of whether the magnetic braking is caused by a coherent (for magnetized shocks) or small-scale (for Weibel shocks) magnetic field. Deceleration of ions together with momentum conservation eventually lead to strong enhancement of the magnetic field in a small region of the shock front. This magnetic field enhancement ensures final thermalization of ions and electrons. Synchrotron emission from electrons from this enhanced magnetic field region seems to be able to explain the observed afterglow emission from gamma-ray bursts, within uncertainty of our knowledge of plasma parameters there.

The proposed mechanism does not exclude the possibility that electrons are heated by strong random electric fields developing due to charge separation in Weibel filaments themselves [19]. Such stochastic heating may be responsible for a significant part of the electron temperature. The question of the relative efficiency of the heating by the cross-shock potential and by the random fields cannot be answered here because of the limitations of our analysis. Further studies are required.

At scales below the ion gyroradius, the most likely scale for Weibel turbulence, differential scattering of ions and electrons by magnetic filaments can cause charge separation and strong electric fields in the shock plane as well as along the shock normal, so it may be nontrivial to distinguish a systematic cross-shock potential from a purely stochastic electric field. Nevertheless, we suggest that a good way to test the idea of a systematic cross-shock potential is to compare the electric field patterns for simulated pair shocks with shock simulations having a realistic ion to electron mass ratio. The mechanism we suggest, which is based upon qualitatively different scattering of electrons and ions, works only for large mass ratios. For pair shocks, on the other hand, electrons and positrons can be separated by small-scale magnetic fluctuations, but there is no systematic charge separation along the shock normal. If the systematic, cross-shock potential drop for electron-ion shocks were comparable to the stochastic component, it would demonstrate the effect we are proposing.

ACKNOWLEDGMENTS

The authors thank Y. Lyubarsky, A. Spitkovsky, and B. Katz for very useful discussions. D.E. was partially supported by the Joan and Robert Arnow Chair in Astrophysics, the Israel-U.S. Binational Science Foundation, and a Center of Excellence grant from the Israel Science Foundation. M.G. and M.A.B. acknowledge support by the Royal Society. M.A.B. acknowledges support by PPARC. M.G. was partially supported by BSF Grant No. 2006095 and ISF Grant No. 275/07.

APPENDIX A: TWO-FLUID HYDRODYNAMICS OF RELATIVISTIC SHOCKS

The basic equations of two-fluid relativistic hydrodynamics read

$$\frac{\partial}{\partial t}n_s + \frac{\partial}{\partial x_i}(n_s v_{s,i}) = 0, \tag{A1}$$

$$\frac{\partial}{\partial t}T_{s,i0} + \frac{\partial}{\partial x_j}T_{s,ij} = n_s q_s (E_i + \epsilon_{ijk} v_{s,j} B_k/c), \tag{A2}$$

where s denotes the species, $i,j,k=1,2,3$, and T_{i0} and T_{ij} are the components of the energy-momentum tensor:

$$T_{0i} = \langle c p_i \rangle, \tag{A3}$$

$$T_{ij} = \langle v_i p_j \rangle. \tag{A4}$$

Here $\langle(\cdots)\rangle$ denotes averaging over the distribution function.

In the one-dimensional stationary case the equations reduce to

$$n v_x = \text{const}, \tag{A5}$$

$$\frac{\partial}{\partial x}T_{ix} = nq(E_i + \epsilon_{ijk} v_{s,j} B_k/c). \tag{A6}$$

These equations should be completed with Maxwell equation with

$$\rho = \sum_s n_s q_s, \tag{A7}$$

$$j_k = \sum_s n_s q_s v_{s,k}. \tag{A8}$$

APPENDIX B: ELECTRON DEMAGNETIZATION

It is known [4] that in narrow nonrelativistic shocks electrons become demagnetized and efficiently heated due to the cross-shock potential. In order to discover whether such demagnetization is possible in relativistic shocks, we reproduce the derivation of Ref. [4] with relativistic corrections. That is, let us assume that a relativistic electron enters an inhomogeneous electric field $E_x = (dE/dx)x$, while the magnetic field inhomogeneity will be neglected. It has been shown [4] that electron demagnetization occurs when two initially close trajectories diverge exponentially. Let us consider two close orbits $x_1(t), y_1(t)$ and $x_2(t), y_2(t)$, each of which is a solution of the equations of motion

$$\frac{d}{dt}(mv_x\gamma) = -eE_x - ev_yB/c, \tag{B1}$$

$$\frac{d}{dt}(mv_y\gamma) = -eE_y + ev_xB/c. \tag{B2}$$

The equations for the differences $\delta x = x_2 - x_1$, $\delta y = y_2 - y_1$, $\delta v_x = v_{2x} - v_{1x}$, and $\delta v_y = v_{2y} - v_{1y}$ can be easily obtained, taking into account that $\delta\gamma = \gamma^3(v_x\delta v_x + v_y\delta v_y)$:

$$\frac{d}{dt}[\gamma(1+\gamma^2v_x^2/c^2)\delta v_x + \gamma^3(v_xv_y/c^2)\delta v_y] = -\frac{e}{m}\frac{dE_x}{dx}\delta x - \Omega\delta v_y, \tag{B3}$$

$$\frac{d}{dt}[\gamma(1+\gamma^2v_y^2/c^2)\delta v_y + \gamma^3(v_xv_y/c^2)\delta v_x] = \Omega\delta v_x, \tag{B4}$$

where we assumed for simplicity that B=const. Here $\Omega = eB/mc$ and E_y=const. In the local approximation the obtained equations are linear equations with constant coefficients and the substitution $\delta x, \delta v_x, \delta v_y \propto \exp(\lambda t)$ gives

$$[\lambda^2\gamma(1+\gamma^2v_x^2/c^2) + (e/m)(dE_x/dx)]\lambda^{-1}\delta v_x = -[\Omega + \lambda\gamma^3(v_xv_y/c^2)]\delta v_y, \tag{B5}$$

$$\lambda\gamma(1+\gamma^2v_y^2/c^2)\delta v_y = [\Omega - \lambda\gamma^3(v_xv_y/c^2)]\delta v_x, \tag{B6}$$

so that eventually

$$\lambda^2\gamma^2(1+\gamma^2v^2/c^2) = -\gamma(1+\gamma^2v_y^2/c^2)(e/m)\frac{dE_x}{dx} - \Omega^2. \tag{B7}$$

The local criterion of instability would read

$$-(e/m)\frac{dE_x}{dx} > \Omega^2/\gamma(1+\gamma^2v_y^2/c^2). \tag{B8}$$

For electrons entering the shock without gyration, $v_y=0$ and $v_x \approx c$, so that one gets trajectory divergence when

$$-\frac{e}{m}\frac{dE_x}{dx} - \frac{\Omega^2}{\gamma} > 0, \tag{B9}$$

with the divergence rate of

$$\lambda = \gamma^{-3/2}\left(-\frac{e}{m}\frac{dE_x}{dx} - \frac{\Omega^2}{\gamma}\right)^{1/2}. \tag{B10}$$

The demagnetized electrons are accelerated by the electric field E_x across the magnetic field up to the point where the demagnetization condition ceases to be satisfied. At this point electrons begin to gyrate and all acquired energy is converted into their gyration energy. Beyond this point the only energy gain is due to the adiabatic conservation of the magnetic moment in the increasing magnetic field.

APPENDIX C: NONLINEAR WAVES AND RELATIVISTIC SOLITONS

We consider a stationary perpendicular wave, $\partial/\partial t = 0$, $\partial/\partial y = \partial/\partial z = 0$, in the framework of the two-fluid hydrodynamics of cold relativistic electrons and protons ($s=e,i$ for electrons and ions, respectively),

$$m_sv_{sx}\frac{d}{dx}(\gamma_sv_{sx}) = q_s(E_x + v_{sy}B_z/c), \tag{C1}$$

$$m_sv_{sx}\frac{d}{dx}(\gamma_sv_{sy}) = q_s(E_y - v_{sx}B_z/c), \tag{C2}$$

$$\gamma_s = (1 - v_{sx}^2/c^2 - v_{sy}^2/c^2)^{-1/2}, \tag{C3}$$

$$n_sv_{sx} = \text{const}, \tag{C4}$$

$$E_y = \text{const}, \tag{C5}$$

$$\frac{dB_z}{dx} = -4\pi\sum_s q_sn_sv_{sy}/c = 4\pi e(n_ev_{ey} - n_iv_{iy})/c, \tag{C6}$$

$$\frac{dE_x}{dx} = 4\pi\sum_s q_sn_s = 4\pi e(n_i - n_e). \tag{C7}$$

It is worth mentioning that in the nonrelativistic limit these equations have the solution in the form of the magnetosonic soliton [14] with the width $\sim c/\omega_{pe}$, where $\omega_{pe}^2 = 4\pi ne^2/m_e$ and the amplitude depends on the Mach number. It should be noted also that $\sim c/\omega_{pe}$ is the dispersion length of linear perpendicular magnetosonic waves.

Following the general principles of [14], we are seeking for nonlinear wave solutions which are asymptotically homogeneous, that is, $n \to n_0$, $v_x \to v_0$, $B_z \to B_0$, and $v_y \to 0$, when $x \to -\infty$. In this case $E_y = v_0B_0/c$. We shall consider weakly nonlinear waves in the sense that deviations from quasineutrality (charge separation) are small throughout the wave profile,

$$\delta n = \frac{1}{4\pi e}\frac{dE_x}{dx} \ll n. \tag{C8}$$

This assumption will be verified *a posteriori*. In this case $n_e = n_i = n \Rightarrow v_{ix} = v_{ex} = v_x$, and $nv_x = n_0v_0$=const. Within this approximation we immediately get

$$m_e\gamma_ev_{ey} + m_i\gamma_iv_{iy} = 0, \tag{C9}$$

$$nv_0v_x(m_e\gamma_e + m_i\gamma_i) + \frac{B_z^2}{8\pi} = nv_0^2(m_e + m_i)\gamma_0 + \frac{B_0^2}{8\pi}, \tag{C10}$$

$$nv_0(m_e\gamma_e + m_i\gamma_i) + \frac{v_0B_0B_z}{4\pi c^2} = nv_0(m_e + m_i)\gamma_0 + \frac{v_0B_0^2}{4\pi c^2}, \tag{C11}$$

where $\gamma_0 = (1 - v_0^2/c^2)^{-1/2}$, and further

$$(m_e\gamma_e + m_i\gamma_i) = (m_e + m_i)\gamma_0 - \frac{B_0^2}{4\pi n_0 c^2}(b-1), \quad \text{(C12)}$$

$$\frac{v_x}{v_0} = \frac{1-\sigma(b^2-1)/2\beta_0^2}{1-\sigma(b-1)}, \quad \text{(C13)}$$

where $\sigma = B_0^2/4\pi n_0(m_i+m_e)c^2\gamma_0$, $\beta_0 = v_0/c$, and $b = B_z/b_0$. It is easy to see that v_x is a monotonically decreasing function of b in the range $1 \leqslant b < 1+1/\sigma$.

Using (C9) and (C6) one obtains

$$\frac{(m_e\gamma_e + m_i\gamma_i)v_{iy}}{m_e\gamma_e} = -\frac{c}{4\pi ne}\frac{dB_z}{dx}. \quad \text{(C14)}$$

In what follows we shall make the assumption that the energy content in ions is always much higher than in electrons, that is,

$$m_i\gamma_i \gg m_e\gamma_e, \quad \text{(C15)}$$

so that approximately

$$m_i\gamma_i v_{iy} = -\frac{cm_e\gamma_e}{4\pi en}\frac{dB_z}{dx}. \quad \text{(C16)}$$

Following the path outlined in the nonrelativistic analysis [14], we substitute (C16) into (C2) for ions to obtain

$$-v_x\frac{d}{dx}\left(\frac{m_e\gamma_e}{4\pi e^2 n}\right)\frac{dB_z}{dx} = v_0B_0 - v_xB_z, \quad \text{(C17)}$$

or, after normalization,

$$\left(\frac{c^2}{\omega_{pe}^2}\right)\frac{1}{N}\frac{d}{dx}\frac{\gamma_e}{N}\frac{d}{dx}b = \frac{(1+\sigma)(b-1)-\sigma b(b^2-1)/2\beta_0^2}{1-\sigma(b-1)}, \quad \text{(C18)}$$

where $N = n/n_0 = v_0/v_x$.

At the asymptotically homogeneous point one has

$$\left(\frac{c^2}{\omega_{pe}^2}\right)\gamma_0\frac{d^2}{dx^2}\xi = (1+\sigma-\sigma/\beta_0^2)\xi, \quad \text{(C19)}$$

where $\xi = b-1 \ll 1$. This point is unstable when $\beta_0^2 > \sigma/(1+\sigma)$, in which case [14] the solution should be of a soliton type (nonperiodic wave). The electron Lorentz factor γ_e cannot be represented as a function of b, so that (C18) cannot be converted to a quasipotential equation. However, we can use the fact that $\gamma_e > 0$ to define a new coordinate $dw = Ndx/\gamma_e$, so that

$$\left(\frac{c^2}{\omega_{pe}^2}\right)\frac{d^2}{dw^2}b = \gamma_e\frac{(1+\sigma)(b-1)-\sigma b(b^2-1)/2\beta_0^2}{1-\sigma(b-1)}. \quad \text{(C20)}$$

The derived equation is valid provided that the flow does not come to a halt, $v_x > 0$, that is,

$$b < b_c = \sqrt{1+2\beta_0^2/\sigma} < 1+1/\sigma. \quad \text{(C21)}$$

When b increases the right-hand side remains positive until $b(b-1) = 2\beta_0^2(1+\sigma)/\sigma$. For $\sigma \ll 1$ and $\gamma_0 \gg 1$ this means that the sign changes when $b = \sqrt{2/\sigma} \gg 1$. At this point the denominator $1-\sigma b \approx 1$. It is well known [15] that there are no soliton solutions for $\sigma \ll 1$ in the pair plasma, where $\gamma_e = \gamma_i = 1-\sigma(b-1)$. For a soliton solution to exist,

$$\int_1^{b_m}\gamma_e\frac{(1+\sigma)(b-1)-\sigma b(b^2-1)/2\beta_0^2}{1-\sigma(b-1)}db = 0 \quad \text{(C22)}$$

has to be satisfied for $b_m < b_c$. Although complete analysis is impossible here it is likely that a soliton solution would not exist for too low σ for the electron-ion plasma as well.

For the analysis of the solution behavior it is sufficient to know that $\gamma_0 \leqslant \gamma_e \lesssim \gamma_i(m_i/m_e)$. It is easy to estimate the typical inhomogeneity scale as $l_s \sim (c/\omega_{pe})\gamma_e^{1/2}$. For $\sigma \ll 1$ (typical for gamma-ray bursts), the highest achievable magnetic field amplitude would grow as $b_{\max} \sim 1/\sigma^{1/2}$, thus ensuring strong magnetic compression. Since $\sigma b \ll 1$ always, the electron current can be estimated as follows:

$$nev_{ey} \sim \frac{n_0ec}{1-\sigma b^2/2}, \quad \text{(C23)}$$

where we assume that electrons remain relativistic: since v_x becomes subrelativistic, $v_{ey} \sim c$. Then the typical length of the magnetic field variation is

$$\left|\frac{B}{(dB/dx)}\right| \sim \frac{B_0b(1-\sigma b^2/2)}{4\pi en_0} \sim \frac{c}{\omega_{pe}}(M\sigma)^{1/2}b(1-\sigma b^2/2), \quad \text{(C24)}$$

where $M = m_i/m_e$. The maximum length is achieved when $b \sim 1/\sqrt{\sigma}$ and $1-\sigma b^2/2 \sim 1$, where $l \sim c/\omega_{pi}$. For smaller $b \sim a/\sqrt{\sigma}$, $a \ll 1$, the length $l \sim a(c/\omega_{pi})$, while for the highest possible $b \sim 1/\sqrt{\sigma}$ and $1-\sigma b^2/2 \sim \sqrt{\sigma}$, and the length becomes $l \sim (c/\omega_{pe})(M\sigma)^{1/2}$.

It has to be understood, however, that the expressions obtained provide only an indication of the character of the wave steepening. Indeed, the strong magnetic compression and narrow width ensure that ions behave nonadiabatically and begin to gyrate strongly in the vicinity of the magnetic field maximum. The ion gyration makes the cold hydrodynamical approximation invalid. Thus, the derived equation (C18) provides a satisfactory estimate of the wave profile only at the upstream edge of the shock ramp [14], which nevertheless is quite sufficient for physical conclusions to be drawn.

Using (C1) one can find

$$\frac{1}{2}\frac{d}{dx}[(m_i^2\gamma_i^2 + m_e^2\gamma_e^2)v_x^2] = -e(m_i\gamma_i + m_e\gamma_e)\frac{d\varphi}{dx}. \quad \text{(C25)}$$

Taking into account the above approximation $m_e\gamma_e \ll m_i\gamma_i$ and the expressions (C12) and (C13), one gets

$$\frac{e\varphi}{m_ic^2\gamma_0} = \int_{b_0}^{b}\left(\frac{\sigma b}{\beta_0^2}\right)\left(\frac{1-\sigma(b^2-1)/2\beta_0^2}{1-\sigma(b-1)}\right)db. \quad \text{(C26)}$$

For the above approximation the potential from the asymptotically homogeneous point to the point where $d^2b/dw^2 = 0$ is easily evaluated as $e\phi \approx 0.5m_ic^2\gamma_0$.

[1] T. Piran, Rev. Mod. Phys. **76**, 1143 (2005); P. Meszaros, Rep. Prog. Phys. **69**, 2259 (2006).
[2] D. Eichler and E. Waxman, Astrophys. J. **627**, 861 (2005).
[3] W. C. Feldman, S. J. Bame, S. P. Gary, J. T. Gosling, D. McComas, M. F. Thomsen, G. Paschmann, N. Sckopke, M. M. Hoppe, and C. T. Russell, Phys. Rev. Lett. **49**, 199 (1982); W. Feldman *et al.*, J. Geophys. Res. **88**, 96 (1983); J. D. Scudder, Adv. Space Res. **15**, 181 (1995).
[4] M. Balikhin, M. Gedalin, and A. Petrukovich, Phys. Rev. Lett. **70**, 1259 (1993); M. Gedalin *et al.*, J. Geophys. Res. **100**, 9481 (1995).
[5] C. C. Goodrich and J. D. Scudder, J. Geophys. Res. **89**, 6654 (1984).
[6] J. A. Newbury *et al.*, J. Geophys. Res. **103**, 29581 (1998).
[7] J. D. Scudder *et al.*, J. Geophys. Res. **91**, 11019 (1986).
[8] M. M. Mellott and W. A. Livesey, J. Geophys. Res. **92**, 13661 (1987).
[9] J. A. Newbury and C. T. Russell, Geophys. Res. Lett. **23**, 781 (1996).
[10] M. F. Thomsen, J. T. Gosling, T. G. Onsager, and C. T. Russell, J. Geophys. Res. **98**, 3875 (1993).
[11] M. Medvedev and A. Loeb, Astrophys. J. **526**, 697 (1999); L. O. Silva *et al.*, Astrophys. J. Lett. **596**, L121 (2003); J. T. Frederiksen *et al.*, *ibid.* **608**, L13 (2004); C. B. Hededal and K.-I. Nishikawa, *ibid.* **623**, L89 (2005); A. Spitkovsky, in *Astrophysical Sources of High Energy Particles and Radiation*, AIP Conf. Proc. No. 801, edited by T. Bulik, B. Rudak, and G. Madejski (AIP, Melville, NY, 2005), p. 345; AIP Conf. Proc. **801**, 345 (2005).
[12] Yu. Lyubarsky and D. Eichler, Astrophys. J. **647**, 1250 (2006).
[13] D. Burgess *et al.*, J. Geophys. Res. **94**, 8783 (1989).
[14] R. Z. Sagdeev, Rev. Plasma Phys. **4**, 23 (1966); M. Gedalin, Phys. Plasmas **5**, 127 (1998).
[15] C. F. Kennel and R. Pellat, J. Plasma Phys. **15**, 335 (1976); D. Alsop and J. Arons, Phys. Fluids **31**, 839 (1988).
[16] J. Newbury *et al.*, J. Geophys. Res. **103**, 29581 (1998); S. Walker *et al.*, Ann. Geophys. **22**, 2291 (2004).
[17] A. Spitkovsky, e-print arXiv:astro-ph/0706.3126v1.
[18] E. Rossi and M. J. Rees, Mon. Not. R. Astron. Soc. **332**, 945 (2003).
[19] C. B. Hededal *et al.*, Astrophys. J. Lett. **617**, L107 (2004); K.-I. Nishikawa *et al.*, Astrophys. J. **622**, 927 (2005); M. Medvedev, Astrophys. J. Lett. **651**, L9 (2006).